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**SEMESTER SYSTEM COURSE  
STRUCTURE  
(CHOICE BASED CREDIT)**

**M. SC. COURSE IN MATHEMATICS  
(PURE AND APPLIED STREAMS)**



**Directorate of Open and Distance Learning  
UNIVERSITY OF KALYANI**

## Semester II

<b>MATCB 2.1</b>	<b>MATC 2.2</b>	<b>MATC 2.3</b>	<b>MATA 2.4 (Elective)</b>	<b>MATP 2.4 (Elective)</b>
Block I	Block I	Block I	Block I	Block I
Block II	Block II	Block II	Block II	Block II
Block III	Block III	Block III		
Block IV				

**MATCB 2.1**  
**Choice Based Paper**  
**Marks : 100**

**Block I**  
**History of Mathematics**  
**Marks : 25 (SEE : 20; IA : 05)**

Units	Topics
1	Babylonian and Egyptian mathematics, Greek mathematics, Pythagoras, Euclid and the elements of geometry, Archimedes, Apollonius,
2	Development of Trigonometry, Development of Algebra, Development of Analytic Geometry
3	Development of Calculus, Development of Selected Topics of Modern Mathematics.
4	Development of Modern geometries, Modern algebra, Methods of real analysis.

**Block II**  
**Operations Research**  
**Marks : 25 (SEE : 20; IA : 05)**

Units	Topics
1	Formulation of linear programming models. Graphical solution. Basic solution (BS) and Basic Feasible Solution (BFS), Degenerate and non-degenerate BFS, Convex set, convex hull, convex polyhedron, extreme points, hyper plane.
2	Standard form of LPP. Simplex method. Charnes' Big – M method.
3	Transportation and assignment problems.
4	A brief introduction to PERT and CPM, Components of PERT/CPM Network and precedence relationships, Critical path analysis.

**Block III**  
**Linear Algebra**  
Marks : 25 (SEE : 20; IA : 05)

Units	Topics
1	Matrix: definition, order, symmetric and skew symmetric matrices.
2	Determinant of a matrix, elementary properties of determinants, inverse of a matrix, normal form of a matrix, rank of a matrix.
3	Elementary concept of a vector space, linear dependence and independence of vectors, basis of a vector space, row space, column space, solution of system of linear equations, Cramer's rule.
4	Eigen values and Eigen vectors of matrices, Cayley Hamilton Theorem, Diagonalization of matrices.

**Block IV**  
**Dynamical Systems**  
Marks : 25 (SEE : 20; IA : 05)

Units	Topics
1	Linearization of dynamical systems: Two, three and higher dimension. Population growth. Lotka-Volterra system.
2	Stability: Asymptotic stability (Hartman's theorem), Global stability (Liapunov's second method).
3	Limit set, attractors, periodic orbits, limit cycles. Bendixon criterion, Dulac criterion, Poincare-Bendixon Theorem. Floquet's theorem.
4	Stability and bifurcation: Routh-Hurwitz criterion for nonlinear systems. Saddle-Node, transcritical and pitchfork bifurcations. Hopf- bifurcation.

**MATC 2.2**  
**Block I**  
**Real Analysis – II**  
**(Pure and Applied Streams)**  
**Marks : 32 (SEE: 25; IA: 07)**

Units	Topics
1	The Lebesgue measure : Definition of the Lebesgue outer measure on the power set of $\mathbb{R}$ , countable subadditivity, Carathéodory's definition of the Lebesgue measure and basic properties. Measurability of an interval (finite or infinite),
2	Countable additivity, Characterizations of measurable sets by open sets, $G_\delta$ sets, closed sets and $F_\sigma$ sets. Measurability of Borel sets, Existence of non-measurable sets.
3	Measurable functions : Definition on a measurable set in $\mathbb{R}$ and basic properties, Simple functions, Sequences of measurable functions, Measurable functions as the limits of sequences of simple functions, Lusin's theorem on restricted continuity of measurable functions, Egoroff's theorem, Convergence in measure
4	The Lebesgue integral : Integrals of non-negative simple functions, The integral of non-negative measurable functions on arbitrary measurable sets in $\mathbb{R}$ using integrals of non-negative simple functions, Monotone convergence theorem and Fatou's lemma.
5	The integral of Measurable functions and basic properties, Absolute character of the integral, Dominated convergence theorem, Inclusion of the Riemann integral, Riesz-Fischer theorem on the completeness of the space of Lebesgue integrable functions.
6	Lebesgue integrability of the derivative of a function of bounded variation on an interval. Descriptive characterization of the Lebesgue integral on intervals by absolutely continuous functions.

**Block II**  
**Complex Analysis – II**  
**(Pure and Applied Streams)**  
**Marks : 32 (SEE: 25; IA: 07)**

Units	Topics
7	Contour integration. Conformal mapping, Bilinear transformation. Idea of analytic continuation.
8	Multivalued functions – branch point. Idea of winding number.
9	Zeros of an analytic function. Singularities and their classification. Limit points of zeros and poles. Riemann's theorem. Weierstrass-Casorati theorem.

10	Theory of residues. Argument principle. Rouché's theorem. Maximum modulus theorem. Schwarz lemma. Behaviour of a function at the point at infinity.
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**Block III**  
**Functional Analysis - II**  
(Pure and Applied Streams)  
Marks : 36 (SEE: 30; IA: 06)

Units	Topics
11	Linear operators, Linear operators on normed linear spaces, continuity
12	Bounded linear operators, norm of an operator, various expressions for the norm. Spaces of bounded linear operators. Inverse of an operator.
13	Linear functionals. Hahn-Banach theorem (without proof), simple applications. Normed conjugate space and separability of the space. Uniform boundedness principle, simple application.
14	Inner product spaces, Cauchy Schwarz's inequality, the induced norm, polarization identity, parallelogram law. Orthogonality, Pythagoras Theorem, orthonormality, Bessel's inequality and its generalisation.
15	Hilbert spaces, orthogonal complement, projection theorem.
16	The Riesz's representation theorem. Convergence of series corresponding to orthogonal sequence, Fourier coefficient, Parseval's identity.

**MATC 2.3**  
**Block I**  
**Mechanics – II (Classical Mechanics)**  
(Pure and Applied Streams)  
Marks : 32 (SEE: 25; IA: 07)

Units	Topics
1	Lagrangian Formulation : Generalised coordinates. Holonomic and nonholonomic systems. Scleronomic and rheonomic systems. D'Alembert's principle. Lagrange's equations. Energy equation for conservative fields. Cyclic (ignorable) coordinates. Generalised potential.

2	Moving Coordinate System : Coordinate systems with relative translational motions. Rotating coordinate systems. The Coriolis force. Motion on the earth. Effect of Coriolis force on a freely falling particle. Euler's theorem. Euler's equations of motion for a rigid body. Eulerian angles.
3	Variational Principle : Calculus of variations and its applications in shortest distance, minimum surface of revolution, Brachistochrone problem, geodesic. Hamilton's principle. Lagrange's undetermined multipliers. Hamilton's equations of motion.
4	Canonical Transformations : Canonical coordinates and canonical transformations. Poincaré theorem. Lagrange's and Poisson's brackets and their variance under canonical transformations, Hamilton's equations of motion in Poisson's bracket. Jacobi's identity. Hamilton-Jacobi equation.
5	Small Oscillations : General case of coupled oscillations. Eigen vectors and Eigen frequencies. Orthogonality of Eigen vectors. Normal coordinates. Two-body problem.

**Block II**  
**Abstract Algebra - II**  
**(Pure and Applied Streams)**  
**Marks: 32 (SEE: 25; IA: 07)**

Units	Topics
6	Preliminaries: Review of earlier related concepts-Rings, integral domains, fields and their simple properties.
7	Detailed discussion on rings: Classification of rings, their definitions and characterization theorem with examples and counter examples. polynomial rings, division algorithm, irreducible polynomials, Eisenstein's criterion for irreducibility.
8	Ideals in rings: Definitions, classifications with related theorems, examples and counter examples
9	Domains in rings: Classification, definitions and related theories with example and counter examples.
10	Field extensions: Definition and simple properties.

**Block III**  
**Operations Research - II**  
**(Pure and Applied Streams)**  
**Marks: 36 (SEE: 30; IA: 06)**

Units	Topics
11	Sensitivity Analysis : Changes in price vector of objective function, changes in resource requirement vector, addition of decision variable, addition of a constraint.
12	Parametric Programming : Variation in price vector, Variation in requirement vector
13	Replacement and Maintenance Models : Failure mechanism of items, General replacement policies for gradual failure of items with constant money value and change of money value at a constant rate over the time period, Selection of best item.
14	Dynamic Programming (DP) : Basic features of DP problems, Bellman's principle of optimality, Multistage decision process with Forward and Backward recursive relations, DP approach to stage-coach problems.
15	Non-Linear Programming (NLP) : Lagrange Function and Multipliers, Lagrange Multipliers methods for nonlinear programs with equality and inequality constraints.
16	Separable programming, Piecewise linear approximation solution approach, Linear fractional programming.

**MATA 2.4**  
**Block - I**  
**Mechanics of Fluids**  
**(Applied Stream)**  
**Marks: 60 (SEE: 50; IA: 10)**

Units	Topics
1	<b>Kinematics</b> : Real and ideal fluids. Streamlines and paths of particles. Steady and unsteady flows. Lagrange's and Euler's methods of description of fluid motion. Accelerations. Boundary surface. Irrotational and rotational motions.
2	Equation of continuity. Equations of Motion : Lagrange's and Euler's equations of motion. Bernoulli's theorem. Cauchy's integrals. Impulsive action.
3	<b>Motion in Two Dimensions</b> : Stream function. Sources, sinks and doublets. Images. Image of a source (sink) with regard to a plane and a sphere.
4	Image of a doublet with regard to a sphere, Images in two dimensions. Milne-Thomson

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circle theorem. Blasius theorem.

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| 5  | <b>General Theory of Irrotational Motion :</b> Flow and circulation. Cyclic and acyclic motions. Impulsive motion. Properties of irrotational motion. Kelvin's theorem of minimum kinetic energy. |
| 6  | Motion of a sphere. Liquid streaming past a fixed sphere. Equations of motion of a sphere.  |
| 7  | <b>Vortex Motion :</b> Vortex motion and its simple properties. Motion due to circular and rectilinear vortices. Vortex pair and doublet. Karman vortex street.                                   |
| 8  | <b>Viscous Liquid Motion:</b> Stress components in real fluid. Rate of strain quadric. Stress analysis in fluid motion.   |
| 9  | Relation between stress and rate of strain. Navier-Stokes' equations.   |
| 10 | Plane Poiseuille and Couette flow between two parallel plates.  |

**Block – II**  
**Stochastic Processes**  
**(Applied Stream)**  
**Marks: 40 (SEE: 30; IA: 10)**

Units	Topics
11	<b>Review of Probability:</b> Random variables, conditional probability and independence, bivariate and multi-variate distributions.
12	Probability generating functions, characteristic functions, convergence concepts.
13	<b>Conditional Expectation:</b> Conditioning on an event, conditioning on a discrete random variable, conditioning on an arbitrary random variable, conditioning on a sigma-field.
14	<b>The Random Walk:</b> unrestricted random walk, types of stochastic processes, gambler's ruin problem, generalisation of the random walk model.
15	<b>Markov Chains:</b> Definitions, Chapman-Kolmogorov equation, Equilibrium distributions, Classification of states, Long-time behaviour. Stationary distribution. Branching process
16	Stochastic process in continuous time: Poisson process and Brownian motion.

**MATP 2.4**  
**Block - I**  
**Differential Geometry - II**  
**(Pure Stream)**  
**Marks: 50 (SEE: 40; IA: 10)**

Units	Topics
1	Curves in the plane and space, surfaces in three-dimension, Smooth surface
2	Tangents and derivatives, normal and orientability, Examples of surfaces.
3	The first fundamental form, Length of curves on surfaces,
4	Isometries of surfaces, Conformal mapping of surfaces,
5	Curvature of surfaces, The second fundamental form, The Gauss and Weingarten map
6	Normal and geodesic curvatures, Parallel transport and covariant derivative.
7	Gaussian, mean and principal curvatures, Gauss Theorema Egregium, Minimal surface
8	The Gauss Bonnet Theorem. Abstract differentiable manifolds and examples, Tangent Spaces

**Block - II**  
**Topology - II**  
**(Pure Stream)**  
**Marks: 50 (SEE: 40; IA: 10)**

Units	Topics
9	Connectedness : Examples, various characterizations and basic properties. Connectedness on the real line.
10	Components and quasi components. Path connectedness and path components.
11	Compactness : Characterizations and basic properties of compactness, Lebesgue, lemma. Sequential compactness
12	BW Compactness and countable compactness. Local compactness and Baire Category Theorem.

- 13 Identification spaces: Constructing a Mobius strip, identification topology, Orbit spaces.
- 14 Some Matrix Lie Groups : Some elementary properties of topological groups.
- 15  $GL(n, R)$  as a topological group and its subgroups.
- 16 Fundamental groups, calculation of fundamental group of S.